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Subject Area	Software

The Sequent Calculus Calculator



Proof written by hand
Own presentation

$\frac{\forall x.P \Rightarrow Q, \neg Q, P \vdash P}{\forall x.P \Rightarrow Q, P \Rightarrow Q, \neg Q, P \vdash \perp}$	$\frac{\forall x.P \Rightarrow Q, P, Q \vdash Q}{\forall x.P \Rightarrow Q, \neg Q, P, Q \vdash \perp}$	hyp	-hyp
$\forall x.P \Rightarrow Q, P \Rightarrow Q, \neg Q \vdash \neg P$	$\forall x.P \Rightarrow Q, P \Rightarrow Q, \neg Q \vdash \neg P$		-goal
$\forall x.P \Rightarrow Q, P \Rightarrow Q \vdash \neg Q \Rightarrow \neg P$	$\forall x.P \Rightarrow Q, P \Rightarrow Q \vdash \forall x.\neg Q \Rightarrow \neg P$		$\forall\text{goal}$
$\forall x.P \Rightarrow Q, P \Rightarrow [x := x]Q \vdash \forall x.\neg Q \Rightarrow \neg P$	$\forall x.P \Rightarrow Q, P \Rightarrow [x := x]Q \vdash \forall x.\neg Q \Rightarrow \neg P$		$[-:]$
$\forall x.P \Rightarrow Q, [x := x]P \Rightarrow [x := x]Q \vdash \forall x.\neg Q \Rightarrow \neg P$	$\forall x.P \Rightarrow Q, [x := x]P \Rightarrow [x := x]Q \vdash \forall x.\neg Q \Rightarrow \neg P$		$[-:]$
$\forall x.P \Rightarrow Q, [x := x]P \Rightarrow Q \vdash \forall x.\neg Q \Rightarrow \neg P$	$\forall x.P \Rightarrow Q, [x := x]P \Rightarrow Q \vdash \forall x.\neg Q \Rightarrow \neg P$		$[-:]$
$\forall x.P \Rightarrow Q \vdash \forall x.\neg Q \Rightarrow \neg P$	$\forall x.P \Rightarrow Q \vdash \forall x.\neg Q \Rightarrow \neg P$		$\forall\text{hyp}$

Proof created using The Sequent Calculus Calculator
Own presentation

Sequent Calculus Calculator
Basic First Order Predicate Calculus With Equality (BasicFoPCE)
Getting started
About

Proof

Please enter a sequent

$$\frac{\forall x.P \Rightarrow Q, \neg Q, P \vdash P}{\forall x.P \Rightarrow Q, P \Rightarrow Q, \neg Q, P \vdash \perp}$$

$$\frac{\forall x.P \Rightarrow Q, P, Q \vdash Q}{\forall x.P \Rightarrow Q, \neg Q, P, Q \vdash \perp}$$

$$\forall x.P \Rightarrow Q, P \Rightarrow Q, \neg Q \vdash \neg P$$

$$\forall x.P \Rightarrow Q, P \Rightarrow Q \vdash \neg Q \Rightarrow \neg P$$

$$\forall x.P \Rightarrow Q, P \Rightarrow Q \vdash \forall x.\neg Q \Rightarrow \neg P$$

$$\forall x.P \Rightarrow Q, P \Rightarrow [x := x]Q \vdash \forall x.\neg Q \Rightarrow \neg P$$

$$\forall x.P \Rightarrow Q, [x := x]P \Rightarrow [x := x]Q \vdash \forall x.\neg Q \Rightarrow \neg P$$

$$\forall x.P \Rightarrow Q, [x := x]P \Rightarrow Q \vdash \forall x.\neg Q \Rightarrow \neg P$$

$$\forall x.P \Rightarrow Q \vdash \forall x.\neg Q \Rightarrow \neg P$$

Proof rule schemas

$$\frac{}{H, P \vdash P} \text{hyp}$$

$$\frac{H \vdash Q}{H, P \vdash Q} \text{mon}$$

$$\frac{H \vdash P \quad H, P \vdash Q}{H \vdash Q} \text{cut}$$

$$\frac{}{H, \perp \vdash P} \perp\text{hyp}$$

$$\frac{H, \neg P \vdash \perp}{H \vdash P} \text{contr}$$

$$\frac{}{H, P \vdash \perp} \text{contr}$$

Instantiation

$$H, ?P \vdash ?P \text{ hyp}$$

Final product
Own presentation

Initial Situation: The sequent calculus is the most widely used style of formal proof in computer science. Its applications include proving logical statements correct and checking the type correctness of programs. The sequent calculus is taught as part of the computer science bachelor curriculum at the HSR.

Problem: Learning the sequent calculus on paper involves copying a lot of large formulae and is therefore tedious. Mechanical checks cannot be performed when proving on paper, which makes the process error-prone. A few online web-based interactive proof assistants already exist. Unfortunately, none of them are currently suitable for use as a didactic aid at the HSR. Given the popularity of the sequent calculus, it would be advantageous to have a well engineered, interactive, web-based proof assistant as a learning aid for the sequent calculus.

Result: This project has resulted in a web-based proof assistant written in a functional programming language that can be used as a didactic aid to teach several different calculi in the sequent calculus style. The solution is extendable and new calculi can be added to the application. Drag and drop style proofs can easily be performed by the web application. The core of the application is independent of the web frontend and can be used with other interfaces.

The thesis shows the possibilities and also the corresponding limits of sequent calculus style proofs that can be performed in web applications.